Nordic Wittgenstein Review 13 (2024) | DOI 10.15845/nwr.v13.3691|[Prepublication for Open Review]

"What Line Can't Be Measured With a Ruler?" Riddles and Concept-Formation in Mathematics and Aesthetics

Samuel J. Wheeler
Sjwheeler(at)uchicago.edu

William H. Brenner
Wbrenner(at)odu.edu

Abstract

We analyze two problems in mathematics – the first (stated in our title) is extracted from Wittgenstein's "Philosophy for Mathematicians"; the second ("What set of numbers is non-denumerable?") is taken from Cantor. We then consider, by way of comparison, a problem in musical aesthetics concerning a Brahms variation on a theme by Haydn. Our aim is twofold: first, to bring out and elucidate the essentially riddle-like character of these problems; second, to show that the comparison with riddles does not reduce their solution to an exercise in bare subjectivity.

1.

In this paper we analyze two problems in mathematics – the first (stated in our title) is extracted from Wittgenstein's "Philosophy for Mathematicians"; the second ("What set of numbers is nondenumerable?") is taken from Cantor. We then consider, by way of comparison, a problem in musical aesthetics concerning a Brahms variation on a theme by Haydn. Our aim is twofold: first, to bring out and elucidate the essentially riddle-like character of these problems; second, to show that the comparison with riddles does not reduce their solution to an exercise in bare subjectivity.[1]

[1] Simo Säätelä, has also taken up the task of elucidating the riddle-like character of certain problems in mathematics. He says: "An open mathematical problem is something that eludes us; it is something that

"'[D]ifficult mathematical problems' […] aren't related to the problem '25 × 25 = ?' as, say, a feat of acrobatics is to a simple somersault (i.e. the relation isn't simply: very easy to very difficult). Rather, they are 'problems' in different senses of the word"[2] (*BT*: 642). Of a straight question we would say: "[It] always refers to a method by which it is solved"[3] (*MWL*: 31). But solving a riddle is like harkening to an inspiration: the riddle seems to *intimate* what answer would make sense of it as a question. "What is always in front of you but never seen?" Answering this requires determining what *x* would turn the word-formation, "*x* is always in front of you but never seen" into a sentence with sense. Using a little imagination, we reply: "The future."[4]

> [A difficult mathematical problem is] like the problem set by the king in the fairy tale who told the princess to come neither naked nor dressed, and she came wearing fishnet. That might have been called not naked and yet not dressed either. [The king's request] was of the form 'Do something which I shall be inclined to call "neither naked nor dressed".' It's the same with a mathematical problem. Do something which I shall be inclined to accept as a solution, though I do not know now what it will be like. (*AWL*: 185–86)

In *Lectures on the Foundations of Mathematics*, Wittgenstein compares the problem of constructing a regular heptagon to "describing the East Pole," the kind of riddle-task for which one would not even be sure where to begin. He goes on to say, "the result of one's search for the construction is that one finds that the question is meaningless"[5] (*LFM*: 64). What happens here is unlike the fairytale in which the king accepts the "wearing of fishnet" as being "neither

we think of as a proposition having a definite truth value, i.e. a definite solution, but we fail to find it and do not even know where to look for it. A problem of this kind has the character of a puzzle or a riddle" (2011: 166). For a sympathetic, yet critical view of this topic, see Schroeder (2007). Two papers we would suggest on a related, but somewhat different topic, are Floyd (2010) and Mühlhölzer (2002).

[2] Cf. "The problem of finding a mathematical decision of a theorem might with some justice be called the problem of giving mathematical sense to a formula" (*RFM:* V, §42) & "'[A] king […] made the law that all who came to his city must state their business and be hanged if they lied. A sophist said he came to be hanged under that law.' What kind of rules must the king give so as to escape henceforward from the awkward position, which his prisoner has put him in? – What sort of problem is this? – It is surely like the following one: how must I change the rules of this game, so that such-and-such a situation cannot occur? And that is a mathematical problem" (*RFM:* VII, §34).

[3] Cf. (*PG*: 377), and Floyd (1995: 393–395).

[4] Cf. (*LFM*: 205–206).

[5] Cf. Baker and Hacker (1985: 300). Also compare: "Like philosophical theses whose sense has not been made determinate, mathematical conjectures, before we possess a method for proving them, do not yet possess a sense […]. Like philosophical problems, mathematical problems assume the appearance of genuine questions when we fail to notice that in framing them, we extend the use of words to contexts in which they have not yet received a determinate meaning." (Nir (forthcoming): 22–23).

naked nor clothed". In *The Realistic Spirit*, Diamond discusses our teaching an infant what 'kissing' is. We start by showing the child how to kiss his toe, then how to *try* to kiss his elbow. Finally, we ask him, "What would it be to kiss your ear?" She goes on to say, "The baby may do something, e.g., may kiss its arm and put the arm to the ear. But what is to count as kissing one's ear was not settled by anything the baby had been taught and whatever is done is not an attempt of the same sort as the attempt to kiss the elbow." This sort task, for which no method of responding has been taught, is, according to Wittgenstein, paralleled in certain difficult mathematical problems.[6] As Diamond goes on to say:

> [T]he training in dealing with requests of this general pattern does not determine what is to count as fulfilling this request […]. We have 'I bisect this angle,' 'I quadrisect this angle,' and so on, and we know how to use these expressions. We then make a new and purely linguistic construction, led to it by analogy with the sentences we have been using: 'I trisect this angle' (*L*, p. 88). But all this is a sentence formed by continuing certain linguistic patterns. We then decide for excellent reasons not to call anything we do 'trisecting an angle' […]. And we might similarly decide there was nothing we wanted to call 'kissing one's ear.' (Diamond 1991: 273)

Similarly, in the case of "What would it be to construct a regular heptagon using only ruler and compasses?" or "to discover the East Pole", the result of searching for an answer to these "questions" would be that we find them meaningless – lacking, indeed, even the "promissory meaning" of riddling-questions at home in our "guessing a riddle" language-games. What we find is that there isn't anything we are going to be willing to call "the construction of a regular heptagon" or the "East Pole."[7]

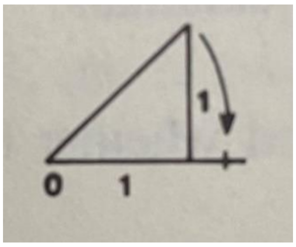
---

[6] Cf. (*LFM*: 17–18): Wittgenstein distinguishes between a physicist who claims to have discovered "how to see what people look like in the dark" or Hardy telling him of a "great discovery [in mathematics]" and being told that "Mr. Smith flew to the North Pole and found tulips all around." The suggestion is that claims like the former, unlike the latter, *mean nothing* independent of knowing *how* these discoveries were made. "For although he speaks English, yet the meaning of what he says depends upon the calculations he has made […]. If I'm told that Mr. Smith flew to the North Pole and found tulips all around, no one would say I didn't know what this meant. Whereas in the case of Hardy I had to know how." In the case of the physicist and Hardy, then, understanding what they say is, like solving a riddle, a matter of making sense of the questions they are attempting to answer and having some understanding of their methods of verification.

[7] Cf. "The proof doesn't *explore* the essence of the two figures, but it does express what I am going to count as belonging to the essence of the figures from now on" (*RFM:* I, §32); "[I]n the riddle-phrase we have something that looks like a description, but what it is for that 'description' to fit something has not been settled" (Diamond 1991: 274). See also (*PI*: §§499–500).

To illustrate how certain mathematical questions can be like riddles, we have taken two concept-forming proofs, reformulating the questions they answer, so that their likeness to riddles becomes obvious. We will then see that the formation of new concepts in these examples is equivalent to the introduction of criteria for something falling under those concepts. The proofs become paradigms, defining, sometimes *picturing,* the concepts they form.[8]

"What line can't be measured with a ruler?"

> [I]s there a method by which we can find a point on a straight line that is not a rational distance from 0? Some have claimed that there is a way of finding such a point – namely by pulling the hypotenuse of a 1-1 triangle down to the base line as such:
>
> 
>
>
> Their idea was that √2 is the result of this construction – whereas really, it is the construction. It's absurd to say that √2 is the length on the base, because the length on the base is what √2 measures. Hence accuracy does not come in, for that has to do with measuring rods. Nor is it an approximation. A construction such as this is a calculation, a symbolism." (AWL: 214–15; slightly reworded).

A difficult passage! We think that what Wittgenstein is getting at here can be explained as follows:

Prior to the introduction of this method for producing a line of a certain length, there was no such thing as "producing a line which is √2 in length."

[8] Before we move on, it should be kept in mind that we will be discussing concept-forming proofs, as opposed to concept-extending or concept-enriching proofs (for a lengthy discussion of this distinction, see Schroeder (2020, Ch. 10)). The latter, we suggest, *may* lack the riddle-like character that the former possess. Further, the comparison of concept-forming proofs to riddles, is just that, a comparison. We are not suggesting that these questions *are* riddles. But rather, that comparing them to riddles may help us better understand the sorts of problems that they are. To this end, we will gladly point out the various differences that are to be found between the two sorts of questions. One such difference comes to light when we consider the fact that the role of concept-forming proofs is to create new concepts. Ordinary riddles, on the other hand, do not introduce new concepts, but play on the different ways in which we use words. The use made of a word in an ordinary riddle is not something *learned* in the process of solving the riddle, but something recollected. Still, both solving a riddle and a mathematical question require that one make sense of the question asked. The fact that *how* one does this – whether by recollecting the different ways in which we do use certain words or by finding a sense for certain expressions – is, in each case, different, does not nullify the *comparison* of mathematical problems to riddles.

The problem, "construct a line which is *exactly* √2, not just approximately, 1.4142135 . . . in length," is like a riddle, in that prior to the introduction of a method for solving it, one is really just "messing about"[9] (Ibid.: 221). That is, a solution to this question is not to be found by reference to our ordinary methods of measurement. After a method is introduced the question gains a definite sense, just as the question, "What runs around a yard but doesn't move?" ("a fence") gains a sense *after* one discerns the way in which the word "run" is being used. From this, however, Wittgenstein goes on to criticize the idea that the above diagram presents for us a base line which is, in fact, √2 in length. This is the *one* line which it makes no sense to call √2 in length. If the diagram *defines* what it is for a line to be that length, then to say that the base line *is* √2 in length would be to try and apply the criterion for a line being such a length *to itself*. Compare this with what Wittgenstein says at *PI* §50:

> There is *one* thing of which one can say neither that it is one metre long, nor that it is not one metre long, and that is the standard metre in Paris. – But this is, of course, not to ascribe any extraordinary property to it, but only to mark its peculiar role in the language-game of measuring with a metre-rule. – Let us imagine samples of colour being preserved in Paris like the standard metre. We define: "sepia" means the colour of the standard sepia which is there kept hermetically sealed. Then it will make no sense to say of this sample either that it is of this colour or that it is not […]. This sample is an instrument of the language used in the ascriptions of colours. In this language-game it is not something that is represented, but is a means of representation.[10] (*PI*: §50)

The above diagram just *is* our criterion for a line being such a length and "It is absurd to say that √2 is the length on the base, for the length is what it measures." Just like the Paris meter, the diagram is not something represented, but is a new means of representation. Further, it is not as if this was the length

[9] Cf. "Wittgenstein in no way attempts to say that creative mathematics is impossible – he does not deny that the mathematical activities inspired or stimulated by 'open problems', even if they can be characterized as a 'messing about', can be mathematically productive, or that they might ultimately lead to a solution of such problems. What he wants to open our eyes to is the misleading classification of these 'open questions' as problems in the same sense as research problems in natural science, or as well-defined questions in mathematics" (Säätelä 2011: 179). Also compare (*PI*: §83) on a game being developed out of playing around with a ball.

[10] Cf. "When I said that a proof introduces a new concept, I meant something like: the proof puts a new paradigm among the paradigms of the language […]. One would like to say: the proof changes the grammar of our language, changes our concepts. It makes new connexions, and it creates the concept of these connexions. (It does not establish that they are there; they do not exist until it makes them.)" (*RFM:* III, §31).

prior to the construction of this method, and we had only ever approximated this with our method of using a ruler. No – "Without the construction √2 is not the length" (*AWL*: 221) *because* the method defines what it is for something to be that length and gives sense to the expression "a line which is √2 in length." This isn't a matter of having ascertained that the line is a certain length with a very precise measure, but a stipulation as to what it means for a line to be that length in the first place.

A comparable but better-known riddle-like question can be found in Cantor:

"What set of numbers is nondenumerable?"

Is there a method by which we can show that a certain set of numbers cannot be put into bijection with the set of natural numbers and thus cannot be enumerated? Georg Cantor argued that there is such a method by way of his diagonal proof.[11]

Cantor is supposed to have shown that there is a method for always producing a new real number, understood as an infinite decimal expansion, *from* the set of all real numbers. The idea was that the diagonal method gives us an example of a set of numbers which is nondenumerable, whereas the method *defines* what it is for a set to be such that its members cannot be put into bijection with the set of natural numbers. That there is a method for always producing a new real number from the set of all real numbers is just *what it means* to be nondenumerable. Cantor did not discover a hidden property of the real numbers but invented a meaning for "a set being nondenumerable". Wittgenstein warns us that "The dangerous, deceptive thing about the idea: 'The real numbers cannot be arranged in a series' […] is that it makes the determination of a concept – concept formation – look like a fact of nature"[12] (*RFM* II, §19).

[11] It is not our intention here to discuss the intricacies of Cantor's proof, as there are many easily available expositions of it to be found. What we do say, however, should not be thought of as an attempt to *down-play* the importance of Cantor's proof. Wittgenstein has often been read as attempting to undermine the proof's impact on mathematics – but this is, in our opinion, a gross misunderstanding. Rather than down-playing the results of the proof, Wittgenstein wants to elucidate precisely what he finds of value in it, namely, the sense in which Cantor's proof arms us with a new concept, 'nondenumerability'. For more extensive discussions of Wittgenstein's views on Cantor, see: Wheeler (2021), Bold (2022), Floyd & Mühlhölzer (2020: 125–191).

[12] Cf. "Can I say: 'A proof induces us to make a certain decision, namely that of accepting a particular concept-formation?'" (*RFM:* IV, §30).

Rather than thinking of Cantor's proof as discerning a fact of nature (or of a Fregean "third realm"), a more fruitful way of talking about the proof, according to Wittgenstein, would be to say something like: "I call number-concept X non-denumerable if it has been stipulated that, whatever numbers falling under this concept you arrange in a series, the diagonal number of this series is also to fall under that concept" (*RFM:* II, §10). This way of expressing the matter makes clear that Cantor's proof is the *determination* of a concept, rather than the description of a concept which comes ready-made.[13]

How, then, are these questions like riddles? In much the same way that solving a riddle is a matter of making sense of the question, so too the mathematical question is given a determinate sense by its proof. As Diamond (1991: 269) puts this: "[I]t is only when one has the solution that one knows how to take the question, what it is for it to have an answer." We might say, Cantor's proof *makes sense* of the question, "What set of numbers is nondenumerable?", just as a riddle-solution makes sense of the original riddle-question. It is not as if, prior to Cantor's proof, we knew what it would be for a set of numbers to be such that they could not be put into bijection with the natural numbers, but we had not yet found the right set of numbers. Rather, Cantor's proof gives determinate sense to this question by stipulating a method for always producing a new Real number from the set of all Real numbers. Likewise, the first proof we discussed makes sense of the question, "What line is √2 in length?" by stipulating a method for constructing a line which is non-rational distance from zero – a task which lacked determinate sense prior to the introduction of this method. The point of similarity between these questions and riddles, then, is to be found in their being *meaning-giving* tasks, rather than questions we already understand prior to our knowing *what it would be* to answer them.[14] Regarding such *meaning-giving* tasks, Juliet Floyd says:

> Wittgenstein likens a mathematical search for the solution to a difficult question […] to trying to wiggle one's ears, without hands, if one doesn't yet know how to;

[13] Cf. "The psychological disadvantage of proofs that construct *propositions* [i.e., concept-forming proofs] is that they easily make us forget that the *sense* of the result is not to be read off from this by itself, but from the *proof*" (*RFM:* III, §25).

[14] Cf. "Looking at the development of Wittgenstein's thought, we could generally say that the later Wittgenstein is less and less interested in a notion of a mathematical proposition as something analogous to the ordinary notion of a proposition. A mathematical 'proposition' does not have a sense prior to its proof. Thus proof becomes essential to the meaningfulness of mathematical formulae, or structures, or questions. We could say that mathematical proof and mathematical sense and truth are mutually dependent" (Säätelä 2011: 173).

> or trying to will an object to move across the room (*WVC*, pp. 34, 136, 144; *PR* XIII; *PG*, p. 393). Before one succeeds or fails, one has no clear understanding of what it would be like to succeed or fail. All the same, one may search for an answer; one 'gropes around,' one tries to do something in the face of one's puzzlement which will generate ('perspicuous') conviction. (Floyd 2000: 249)

Further, just as coming up with an answer to the riddle, "What can be kept even after it is given to someone?" ("one's word") calls for imagination in determining what we are going to call "that which can be both given and kept," so too these concept-forming proofs call for imagination in determining what it makes sense to us to call "a set which is nondenumerable" or "a line which is √2 in length." As Wittgenstein says: "A mathematical question is a challenge. And we might say: it makes sense […] if it stimulates the mathematical imagination"[15] (*Z*: §§696–697).

2.

The two foregoing problems exemplify the riddle-like character attributable to certain questions in mathematics. Wittgenstein recognized this: "You say 'Where there is a question there is also a way to answer it, but in mathematics there are questions that we don't see any way to answer.' – Quite right, but all that follows from that is that in this case we are using the word 'question' in a different sense" (*BT*: 642) To highlight this very different sense of "question," Wittgenstein compares problems in mathematics to those in aesthetics. (*CV*: 25) In aesthetics, an explanation is not just a matter of stating facts, but rather, of showing someone how to see the facts in a new way. To be convinced of something in aesthetics, as in mathematics, is to see both *that* it is the case, and *how* it is so. "The question of aesthetics is not 'Do you like this?' but 'Why do you like it?'"[16] (*PO*: 105). As Säätelä (2011: 176) aptly points out,

> [T]he conviction we reach when understanding a mathematical proof is of the same kind as the conviction an 'aesthetic investigation' can produce – here as well the

[15] Cf. "I may let a formula stimulate me. Thus I shall say, Here there is a *stimulus* – but not a *question*. Mathematical 'problems' are always such stimuli" (*WWK*: 144).

[16] Cf. "Proof, one might say, does not merely shew *that* it is like this, but: *how* it is like this. It shows *how* 13 + 14 yield 27" (*RFM:* III, §22); "Imagine a sequence of pictures. They shew how two people fence with rapiers according to such-and-such rules. A sequence of pictures can surely shew that. Here the picture refers to a reality. It cannot be said to shew *that* fencing is done like this, but *how* fencing is done. In another sense we can say that the pictures shew how one can get from this position into that in three movements. And now they also shew *that* one can get into that position in this way" (*RFM:* V, §51). See also Diamond (1991: 270).

> reasons or grounds that are presented have to show not only that something is what it is, but also why or how it is like it is. That is, it is not enough for someone to tell me there is a solution; I must also be given reasons to accept the solution.

To make the analogy with aesthetics concrete, we will look at an example of a musical riddle from Wittgenstein:

> Take a theme like that of Haydn's (St. Antony Chorale), take the part of one of Brahms's variations corresponding to the first part of the theme, and set the task of constructing the second part of the variation in the style of its first part. That is a problem of the same kind as mathematical problems are. If the solution is found, say as Brahms gives it, then one has no doubt; – that is the solution.[17] (*RFM:* VII, §11)

Note that it is not because *Brahms* gave it that it is the solution. To be the solution it has to be generally acknowledged as such by the appropriate musical community. "What belongs to a language game is a whole culture" (*LA*: 8). In a remark from 1946, Wittgenstein discusses the sense in which a repeat may be "*necessary*" in a theme, and criticizes the idea that there is some "model [which] already exists in reality" but for which "the theme only approaches it, corresponds to it" if the section *is* repeated. What he wants to criticize is the idea that there is something, *outside* the theme itself and our life with music, which is a paradigm to which the theme must correspond. But the only paradigm apart from the theme, he suggests, would be "the whole range of our language games," or "the rhythm of our language, and our thinking and feeling"[18] (*CV*: 52). So too, our accepting something as a variation on a theme,

---

[17] Here, Wittgenstein is comparing the different ways one may take in developing a language that avoids Russell's paradox with the various ways one may develop a variation on a musical theme. Also comparable, we suggest, are the various ways one may take in developing an answer to a moral dilemma – as in Sartre's example of the young Frenchman during the Nazi occupation faced with the question: "Should I join the Resistance, or take care of my elderly mother?" Both of these answers made sense *as* solutions to the problem – unlike, e.g., "become a ski instructor!" His problem was an ethical riddle, rather than a problem with a standard method of solution. In a case such as this, says Robert Fogelin, where a man's "filial duties come into conflict with patriotic duties," the man "has no choice other than to act on his own and in the process define his moral character" (Fogelin 2003: 63). And here, *that is* the solution – cf. Wittgenstein's analysis of a moral dilemma (in Rhees 1965: 22).

[18] Cf. Fogelin (2003: 153–155). Consider, as an analogous point, Wittgenstein's discussion of our saying that "the steps are determined by the formula" (*RFM:* I, §1). Rather than *rejecting* that the steps of a series are determined by a formula, Wittgenstein wants to show that this logical necessity is not something *outside* or *beyond* our mathematical practice, but is bound up with our life with mathematics. The examples he suggests then, of a series being "determined," are quite ordinary: from "dictating to a student the series she is to write down" to "writing the series faintly and having the student trace over it" (*RFM:* I, §22).

or as the solution to a riddle, is not done by appealing to some standard of correctness outside of our lives with music or language. Remember, making-up and guessing riddles is a language-*game*, not just arbitrary playing around with signs (*PI:* §23). In spite of its being a *guessing* language-game (in contrast to, "constructing an object from a description," for instance), it is not a mere playing around with words, where "whatever is going to seem right to me is right" (*PI*: §258).[19] So too, composing a variation may be called a language-*game*, not just arbitrary playing around with a theme. Suppose, however, another musician *did* doubt Brahms' solution. One might get her to "hear" the second part of the variation as indeed in the style of the first part, perhaps by showing her lots of different pieces by Brahms.[20] If she is persuaded to accept Brahms' solution it will be by some such informal approach – *not* by showing her that what Brahms did was correct because it corresponded to a preexistent standard of correctness.[21] As "a mathematician is of course guided by associations, by certain analogies with the previous system," so too a composer is guided by associations and analogies with previous compositions.[22] (*WWK*: 144) And just as we might say of a character in a novel that she does not fit into the story, so too, an attempt at composing a variation may be rejected for failing to fit into the context of the original theme.[23] For both the mathematician and the composer, their activity "is carried on in a particular

---

The perspective that Wittgenstein wants us to take up in regard to this picture is analogous to the sense in which we might speak of the steps in a practice as determined by the training which underlies it. "The steps are all already determined by the formula" – that is, determined *by this being a practice which we teach with strictness and rigor*, of which we expect general agreement.

[19] Cf. (*PI*: §337), where Wittgenstein points out that the language-game of 'intending something' is "embedded in its situation, in human customs and institutions."

[20] Cf. "Brahms *reason* for rejecting Joachim's suggestion that his Fourth Symphony should be opened by two chords was not that that wouldn't produce the feeling he wanted to produce, but something more like 'That isn't what I meant.'[…]. [Y]ou can make a person see what Brahms was driving at by showing him lots of different pieces by Brahms, or by comparing him with a contemporary author" (*PO*: 106) & "I hear a melody completely differently after I have become familiar with its composer's style. Previously I might have described it as happy, for example, but now I sense that it is the expression of great suffering. Now I describe it differently, group it with quite different things" (*LW*-I: §774).

[21] Cf. "Soulful expression in music – this cannot be recognized by rules" (*Z*: §157). See also (*PI*-II: 227) on "expert judgment."

[22] Cf. "A creative mathematician understands an unproven mathematical proposition only as a composer understands a theme that he has resolved to integrate into an existing composition" (Baker and Hacker 1985: 299).

23 Cf. (CV: 57). Also compare: "I read a sentence from the middle of a story […]. Do I understand the sentence? […] It is an English sentence, and to that extent I understand it. I should know how the sentence might be used, I could invent a context for it. And yet I do not understand it in the sense in which I should understand it if I had read the story." (PG: 43).

sphere [and even] when a question is asked for which there is no [given] method of answering it, we do know certain requirements the answer must fulfill" (*AWL*: 221–22). It is a mark, then, of both composing a variation and solving a riddle, that one cannot appeal to ordinary standards of correctness in justifying one's composition or riddle-solution. What one must do is to show how what one has done should be seen as a variation on a theme or as a satisfactory riddle-solution. Just as one might justify a composition by getting someone to "hear" the variation as pertinently connected to the original theme, so too, solving a riddle is a matter of getting someone to "hear" the riddle-solution *as* a solution.

While not "straight questions", neither the musical nor the mathematical questions are *meaningless*. "[T]he mathematician can be said to 'understand' [the mathematical conjecture] only in the sense in which he has an inkling of what kinds of techniques might be employed in its proof, a hunch as to the ways in which it might be woven into the fabric of mathematics" (Baker & Hacker 1985: 300).[24] Their meaning is in part determined by the fact that we may rule out certain solutions from the outset, depending on the particular "sphere" in which they occur – just as we can rule out certain variations or solutions to ordinary riddles from the outset. To the question, "What has to be broken before you can use it?" ("an egg") we can be certain, because of the sorts of activities that we engage in, and the specific form of life we inhabit, that the solution will not be "a baseball bat".[25]

In "Riddles and Anselm's Riddle", Cora Diamond imagines someone asking, "What's green, hangs on the wall, and flies?" (Diamond 1991: 272). The suggestion is that such a question isn't deserving of the name "riddle" since there is not a sphere, practice, or form of life in which one suggestion for

[24] Cf. "Now isn't it absurd to say that one doesn't understand the sense of Fermat's last theorem? — Well, one might reply: the mathematicians are not *completely* blank and helpless when they are confronted by this proposition. After all, they try certain methods of proving it; and, so far as they try methods, *so far* do they understand the proposition. — But is that correct? Don't they *understand* it just as completely as one can possibly understand it? […] 'Understanding' is a vague concept." (*RFM:* VI, §13).

[25] Cf. "[S]uppose that Rachmaninov had inserted 'God Save the Queen' as one of the variations on [a] theme by Paganini and, when questioned about this, he said that he first heard Paganini's theme during a concert at which the Queen of England was present. We should not accept, on this ground, that he had written a variation…[W]hat makes something a correct step in composing a variation…is that it is connected pertinently with what precedes it; what makes a connection pertinent is settled by the reactions of the practitioners themselves" (Mounce 1981: 116–119).

answering it would strike us as more plausible or less plausible than another.[26] Of course, "knowing one's way about" in the sphere of thought in which the riddle is posed does not guarantee success in answering it. There is, indeed, "no exact way of working out a solution. One can only say, 'I shall know a good solution if I see it.'" (*LFM*: 84).

***

We end with a few words about musical and mathematical riddles. Solving mathematical riddles will *often* produce new forms of description and our "accepting a particular concept-formation" (*RFM:* IV, §30), while music may tend to enrich or extend the concepts we already know how to use, such as "question and answer," "joy", "fear", "love", or "humor".[27] As an example, prior to a recent performance of Beethoven's Violin Sonata No. 8 in G Major, the pianist advised the audience to listen closely for the "humor" in the piece. While listening, I was so struck by the humor, even though, prior to this performance, it had never occurred to me what it might be for a phrase of music to be humorous.[28] This experience, I want to say, constituted an *extension* of my concept "humor".[29]

[26] Cf. (*PGL*: 32–33). Lytton Strachey wrote a description of Queen Victoria's last thoughts. The story of her dying thoughts, according to Wittgenstein, is meaningful only in connection to her entire life, and cannot be understood in isolation; they can only be understood as the culmination of that life. Only within this context are there answers which would strike us as more plausible or less plausible than others – and others which we would rule out entirely. (Cf. Diamond (191: 239) on imagining Hitler's dying thoughts.)

[27] This is, of course, a very complex situation, and we are not saying that *all* mathematical proofs form new concepts, nor that our talk about music only brings about the extension of ordinary concepts. Concept-extending proofs, we think, are both important and prevalent in mathematics, just as concept-formation may play an important role in music. Consider, for example, Schroeder's discussion (2020: Ch. 10) of Euclid's proof that there are an infinite number of prime numbers, Wittgenstein's discussion (*AWL*: 217) of the extension of our concepts "comparable" and "smaller" when we move from comparing two rationals such as 3 and ½ to comparing a rational number and an irrational such as √2, as well as concepts like "rhythm", "tempo", and "harmony", which seem to have their primary application in our music-talk.

[28] Cf. "Remember the impression one gets from good architecture, that it expresses a thought" (*CV*: 22); "Understanding a musical phrase may also be called understanding a *language*" (*Z*, §172); "It was said of Labor's playing: 'He is *speaking*.' How curious! […] Music, *some* music at least, makes us want to call it language; but some music of course doesn't" (*CV*: 62). What we see here is the *extension* of our concepts "speech" and "language".

[29] We would like to express our gratitude to everyone who supported us and read this paper in the various stages of its development. This includes Zachary Tabor and Michael Tarpey, as well as Jim Conant, Felix Mühlhölzer, Severin Schroeder, and Philip Bold. Many thanks also to the anonymous reviewers for their insightful suggestions which made this a better paper.

References

Baker, G. and Hacker, P.M.S., 1985. *An Analytical Commentary on* The Philosophical Investigations, Vol. 2. Oxford: Blackwell.

Bold, P., 2022. *Three Essays on Later Wittgenstein's Philosophy of Mathematics: Reality, Determination, & Infinity*. PhD diss. University of North Carolina at Chapel Hill.

Diamond, C., 1991. *The Realistic Spirit*. Cambridge, MA: MIT Press.

Floyd, J., 1995. "On Saying What You Really Want To Say: Wittgenstein, Gödel, and the Trisection of the Angle". In: *Essays on the Development of the Foundations of Mathematics*, ed. J. Hintikka, 373–425.

Floyd, J., 2000. "Wittgenstein, Mathematics and Philosophy". In: *The New Wittgenstein*. ed. A. Crary & R. Read. New York, NY: Routledge, 232–261.

Floyd, J., 2010. "On Being Surprised: Wittgenstein on Aspect-Perception, Logic, and Mathematics". In W. Day & V.J. Krebs (eds.), *Seeing Wittgenstein Anew*. Cambridge: Cambridge University Press.

Floyd, J. and Mühlhölzer, F., 2020. *Wittgenstein's Annotations to Hardy's Course of Pure Mathematics*: *An Investigation of Wittgenstein's Non-Extensionalist Understanding of the Real Numbers*. Cham, Switzerland: Springer Publishing.

Fogelin, R., 2003. *Walking the Tightrope of Reason: The Precarious Life of a Rational Animal*. New York, NY: Oxford University Press.

Mounce. H.O., 1981. *Wittgenstein's Tractatus: An Introduction*. Chicago, IL: The University of Chicago Press.

Mühlhölzer, F., 2002. "Wittgenstein and Surprises in Mathematics". In: R. Haller and K. Puhl, eds. *Wittgenstein and the Future of Philosophy: A Reassessment after 50 Years*. Vienna: öbv&hpt Verlagsgesellschaft, 306–315.

Nir, G., (forthcoming). "Nonsense: A Riddle Without Solution".In: *Early Analytic Philosophy: Origins and Transformations*, ed. J. Conant & G. Nir.

Rhees, R., 1965. "Some Developments in Wittgenstein's View of Ethics". *Philosophical Review* 74, 17–26.

Säätelä, S., 2011. "From Logical Method to 'Messing About': Wittgenstein on 'Open Problems' in Mathematics". In: *The Oxford Handbook of Wittgenstein*, ed. O. Kuusela & M. McGinn. Oxford: Oxford University Press, 162–180.

Schroeder, S., 2009. "Conjecture, Proof, and Sense in Wittgenstein's Philosophy of Mathematics". In: C. Jäger and W. Löffler (eds.), *Epistemology: Contexts, Values, Disagreement*. Berlin: De Gruyter.

Schroeder, S., 2020. *Wittgenstein on Mathematics*. New York, NY: Routledge.

Wheeler, S., 2021. "Defending Wittgenstein's Remarks on Cantor from Putnam". *Philosophical Investigations* 45, 320–333.

Wittgenstein, L., 1953. *Philosophical Investigations* (Third Edition). Trans. G.E.M. Anscombe. New York, NY: Macmillan Publishing Co., Inc. (*PI*)

Wittgenstein, L., 1966. *Lectures and Conversations on Aesthetic, Psychology, and Religious Belief*. Ed. C. Barrett. Berkeley, CA: University of California Press. (*LA*)

Wittgenstein, L., 1967. *Zettel*. Ed. G.E.M. Anscombe and G.H. von Wright, trans. G.E.M. Anscombe. Oxford, UK: Basil Blackwell. (*Z*)

Wittgenstein, L., 1974. *Philosophical Grammar*. Ed. Rush Rhees, trans. Anthony Kenny. Oxford, UK: Basil Blackwell. (*PG*)

Wittgenstein, L., 1976. *Lectures on the Foundations of Mathematics*. Ed. C. Diamond. Chicago, IL: The University of Chicago Press. (*LFM*)

Wittgenstein, L., 1978. *Remarks on the Foundations of Mathematics*. Ed. G.H. von Wright, R. Rhees, G.E.M. Anscombe, trans. G.E.M. Anscombe. Cambridge, MA: The MIT Press. (*RFM*)

Wittgenstein, L., 1979a. *Wittgenstein and the Vienna Circle*. Ed. B. McGuinness, trans. J. Schulte and B. McGuinness. Oxford, UK: Basil Blackwell. (*WWK*)

Wittgenstein, L., 1979b. *Wittgenstein's Lectures: Cambridge 1932–1935*. Ed. A. Ambrose. Chicago, IL: The University of Chicago Press. (*AWL*)

Wittgenstein, L., 1980. *Culture and Value*. Ed. G.H. von Wright, trans. P. Winch. Chicago, IL: The University of Chicago Press. (*CV*)

Wittgenstein, L., 1982. *Last Writings on the Philosophy of Psychology, Volume I*. Ed. G.H. von Wright & Heikki Nyman, trans. C.G. Luckhardt & M.A.E. Aue. Chicago, IL: The University of Chicago Press. (*LW*-I)

Wittgenstein, L., 1988. *Wittgenstein's Lectures on Philosophical Psychology, 1946–47*. Ed. P.T. Geach. Chicago, IL: The University of Chicago Press. (*PGL*)

Wittgenstein, L., 1993. *Philosophical Occasions, 1912–1951*. Ed. J. Klagge and A. Nordman. Indianapolis, IN: Hackett Publishing Co. (*PO*)

Wittgenstein, L., 2013. *The Big Typescript: TS 213*. Ed. and trans. C.G. Luckhardt and M.A.E. Aue. Oxford, UK: Blackwell. (*BT*)

Wittgenstein, L., (Unpublished MS). "Ludwig Wittgenstein on 'Personal Experience' Macdonald Notes, 1935–1936." Ed. C. Diamond. (*MWL*)

Biographical Note

Samuel J. Wheeler is currently a PhD student in Philosophy at the University of Chicago, and received his BA in Philosophy from Old Dominion University. His primary interest is in Wittgenstein, with an emphasis on the philosophy of mathematics, religion, and mind, as well as the intersection between philosophy and literature. He is the author of "Defending Wittgenstein's Remarks on Cantor from Putnam" and "Wittgenstein on Miscalculation and the Foundations of Mathematics", both published in *Philosophical Investigations*. Other publications include "A Response to Dehnel", and a review of Severin Schroeder's book, *Wittgenstein on Mathematics*.

William H. Brenner is Professor Emeritus of Philosophy & Religious Studies at Old Dominion University. He received his PhD from the University of Virginia, and his BA from the College of St. Thomas. His publications include *Logic & Philosophy: An Integrated Introduction and Wittgenstein's Philosophical Investigations*, together with a number of articles – notably, "Chesterton, Wittgenstein, and Foundations of Ethics", "'The Soulless Tribe'", "Natural Law, Motives, and Freedom of the Will", "Dialogues on Causality and the Limits of Empiricism", "'Theology as Straw': An Essay on Wittgenstein and Aquinas", "Wittgenstein's 'Kantian Solution to the Problem of Philosophy'", and "From Inverted Spectra to Colorless Qualia".